Math 458C Second Exam. Dec. 9 2009 Name: ________________________________

Each problem is worth 25 points. In calculations, give three or more digits after the decimal point. Show all work. Correct answers with incorrect or incomplete work will not receive full credit.

1. Suppose that \( X(t) \) follows the process \( dX = 3X \, dt + 2X \, dz \) where \( z \) is a standard Wiener process. Using Ito’s Lemma, find the process for \( Y(t) = X(t)^4 \). 

2. Create a two-month binomial lattice model to find the value of a European put option with strike price \( K = 55 \), initial stock price \( S(0) = 54 \), volatility \( \sigma = 0.33 \), and annual interest rate \( r = 0.12 \). Use \( \Delta t = 1/12 \) yr.

3. An investor buys 10,000 calls with strike \( K_1 = 60.00 \) for \( C = 1.39 \) each and sells 10,000 calls with strike \( K_2 = 62.50 \) for \( C = 0.26 \) each. Sketch the payoff and profit curves for this spread. Show the break-even point(s) where the profit is zero. If you are the investor, what do you hope the stock will do?

4. a) Show that for any constant \( K \), the function \( f(S,t) = Ke^{-r(T-t)} - S \) is a solution of the Black-Scholes equation.

b) Suppose we know that \( C(S,t) \), the price of a European call, is a solution of the Black-Scholes equation. Let \( P(S,t) \) be the price of a European put. Use the put-call parity formula \( P(S,t) = C(S,t) + Ke^{-r(T-t)} - S \) together with the result from part a) to show that \( P(S,t) \) is also a solution of the Black-Scholes equation.