3.4.1 Shocks, entropy condition
\[ u_t + \left( \frac{k^2}{2} \right)_x = 0 \]

Now \[ u(x, 0) = g(x) = \begin{cases} 0 & x < 0 \\ 1 & x > 0 \end{cases} \]

Could draw:

\( x = t/2 \)

\( u = 0 \)

\( u = 1 \)

\( 0 = \frac{x}{t} \)

\( u = 1 \)

\( u \) is continuous; no curves crossing.
Under some conditions such as the above, need an additional constraint on set of solutions, to obtain a unique solution.

Idea: if follow characteristics backwards in time, do not expect to encounter curves of discontinuity. If forwards in time, must be "compression".

Let $C$ be curve of discontinuity

\[ u = g(s) = g(x^0) \text{ on curve } C. \]

\[ (x_1(s), x_2(s)) = \left( F'(g(x^0)) s + x^0, s \right) \]

\[ \text{For compression, } \frac{F'(g(x^0))}{u_0} u > C > \frac{F'(g(x^0))}{u_n} u_n \]

\[ u = g(s) = g(x^0) \text{ increasing at rate } F'(g(x^0)) \text{ at } x \]

\[ \kappa = \kappa(s) \text{ is increasing at rate } F'(g(x^0)) \text{ at } x \]

\[ u = g(s) = g(x^0) \text{ at rate } F'(g(x^0)) \text{ at } x \]

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