Problem 2.5.1.

For any point \((x, t)\) in \(\mathbb{R}^n \times (0, \infty)\). For any self-defining \(Z(s) = U(x + bs, t + s)\). Then, from the rule, we have

\[
\dot{Z}(s) = b \cdot D_x U_t + U_t = -C U(x + bs, t + s) = -C Z(s)
\]

Above, \(Z(s) = ke^{-Cs}\), for some constant \(k\).

We can solve for \(k\) by letting \(s = -t\), then

\[
Z(-t) = U(x - bt, 0) = g(x - bt).
\]

So,

\[
Z(-t) = ke^{ct}.
\]

Therefore,

\[
Z(s) = g(x - bt) e^{-ct(s)} = U(x + bs, t + s)
\]

Hence, \(U(x, t) = g(x - bt) e^{-ct}\), when \(s = 0\).