12.6 Basics of Futures Contracts
Futures market: accounts for price fluctuations, so pays immediately.

Everyone who trades, has a margin account.

Every day, if a price change leads to a gain, the gain is added to the margin account.

If a price change leads to a loss, the loss is taken off the margin account.

If price today is $5.23/unit of commodity and I hold 1000 units; and tomorrow price changes to $5.25/unit, then I get $(5.25 - 5.23) \times 1000 = $20 added to my margin account.
maintenance level:

If margin account goes below this level,
must either 1. close out position & accept the loss
    2. replenish margin account
to required maintenance level.

If account rises above maintenance level,
can take out extra.

If there is a delivery (the day in mid-July arrives)
the futures price is the price on the delivery day,
and gain/loss is (already) in margin account.

Process of adjusting margin accounts
is called "marking to market"
Let $F$ be forwards prices, $F$ be futures prices.

Consider two strategies, B and A.

Strategy B

At $t = 0$, arrange a forward contract;
agree to purchase 1 unit of asset at $T = M + t$,
for $F_0$.

At $t = T$, profit = $S(T) - F_0$

↓

spot price of commodity/asset at $T$. 
Strategy A

\[ t = 0 \at \quad t = M \Delta t \]

at \( t = 0 \), go long \( d_{1m} \) units of future, for delivery at \( T = M \Delta t \).

(purchase)

at \( t = t_1 = \Delta t \), price changes from \( F_0 \) to \( F_1 \).

margin account is changed by \( (F_1 - F_0) d_{1m} \)

go long \( (d_{2m} - d_{1m}) \) units of future, same delivery \( T \).

now hold \( d_{1m} + (d_{2m} - d_{1m}) = d_{2m} \) units of future.

\[ \begin{cases} \text{if } F_1 > F_0, & \text{take } (F_1 - F_0) d_{1m} \text{ from margin account} \text{ to invest in bank;} \\ \text{if } F_1 < F_0, & \text{borrow } (F_0 - F_1) d_{1m} \text{ from bank to add to margin account.} \end{cases} \]
Strategy A (std).

at \( t = t_2 = 24t \): price changes from \( F_1 \) to \( F_2 \),

margin account is charged by \( $(F_2 - F_1) d_2 \).

4 go long \( (d_3 - d_2) \) units of future, same delivery.

now hold \( d_3 + (d_3 - d_2) = d_3 \) units of future.

Continue until \( t = t_{M-1} = (M-1) \Delta t \).

\[
\begin{array}{c|c|c|c|c}
0 & (F_1 - F_0) d_0 & (F_2 - F_1) d_1 & (F_3 - F_2) d_2 & \cdots & (F_M - F_{M-1}) d_{M-1} \\hline
\end{array}
\]

value of cash to bank at \( t = M \Delta t \)

is:
\[
(F_1 - F_0) d_0 + \cdots + (F_2 - F_1) d_1 + \cdots + (F_M - F_{M-1}) d_{M-1} = -F_0 + 0 + c + \cdots + F_M = S(T)
\]

\( C = S(T) - F_0 \).
If $F_0 \neq F_0$, there is an arbitrage opportunity by combining strategies A and B.

Theoretically $F_0 = F_0$. 