12.12 Hedging Nonlinear Risk
Nonlinear risk.

"Corn" is grown by a large number of producers; say 100 farmers. Amount grown by each, highly correlated with all of the others.

Suppose each produces \( c \) bushels: total 1000 bushels.

Price
\[ \text{demand curve}. \]
\[
\begin{array}{c|c|c}
\text{Price} & \text{Demand} \\
$10 & 500,000 \\
$5 & 1,000,000 \\
\end{array}
\]

\[ P = P_d(D) \]
\[ P_d(D) = 10 - \frac{D}{10^5} \]

If: total demand = total production, then \( D = 100c \).

Revenue (each farmer) is
\[ R_p(c) = P_d(100c) \cdot c = \left(10 - \frac{c}{10^3}\right) \cdot c \]
\[ R_p(c) = 10 - \frac{C}{10^3} = 0 \Rightarrow C = \frac{10^3}{2} = 5000. \]

Suppose expected production is \( \bar{C} = 5000 \).

For \( C \) near \( \bar{C} \), \( R_p'(c) > 0 \)

Farmer has a "natural hedge" against decreasing prices; provided \( C < 5000 \), namely increasing production \( \Rightarrow \) revenue increase.

If \( C > 5000 \), however, if prices decrease, then despite increasing production revenue is decreasing.
Consider an effort to counteract these effects, by using futures contracts.

Each future is a contract to buy 1 bushel of corn at price \( \bar{p} \), at time \( T \) in future.

Suppose farmer arranges \( h \) such futures:

\( h > 0 \) is buy \( h \) bushels for \( \bar{p} \) at \( T \)

\( h < 0 \) is sell \( |h| \) bushels for \( \bar{p} \) at \( T \)

Payoff from futures:

\[ h(P - \bar{p}), \quad P = P(T) = \text{spot price at time } T. \]

Total revenue:

\[ R_{\text{tot}} = R_p(C) + h(P - \bar{p}) = \left(10 - \frac{C}{10^3}\right)C + h(P - \bar{p}). \]
\[ R_{\text{tot}}(C, h) = 10C - \frac{C^2}{10^3} + h(P - \overline{P}) \]

But \( P(T) = P_d(100C) \) and \( \overline{P} = P_d(100\overline{C}) \),

\[ \Rightarrow P(T) - \overline{P} = P_d(100C) - P_d(100\overline{C}) \]

\[ = 10 - \frac{100C}{10^5} - (10 - \frac{100\overline{C}}{10^5}) \]

\[ = \frac{1}{10^3} (\overline{C} - C) \cdot \frac{C}{10^3} + \frac{h}{10^3} (\overline{C} - C). \]

\[ R_{\text{tot}} = 10C - \frac{C^2}{10^3} + \frac{h}{10^3} (\overline{C} - C). \]

\[ \frac{\partial R_{\text{tot}}(C, h)}{\partial C} = 10 - \frac{2C}{10^3} + \frac{h}{10^3} (-1) = 0 \text{ for } h = 10^3 \left(10 - \frac{2\overline{C}}{10^3}\right). \]

For \( \overline{C} = 3000 \),

\[ \frac{\partial}{\partial C} R_{\text{tot}}(C, h) \bigg|_{C=\overline{C}} = 0 \text{ when } h = 10^3 \left(10 - \frac{2\times3000}{10^3}\right) = 4000. \]

\[ C = \overline{C} \]

buy 4000 bushels!
Links visited in class

"Corn" producer hedges with futures nonlinear risk,