11.6 Linear Pricing
11.6 Arbitrage Example (Actual event, Spring 2016)

$CDN \rightarrow US$ exchange rate in bank is: $1 CDN = 0.70 US$

NYS Thruway: considers $5 CDN (bill) = 4.70 US$

Motorist:
- travels Wonsville $\rightarrow$ Depew; gives $5 CDN$ to collector at Depew.

Collector: returns $4.70 US - 0.15 US = 3.85 US$

Motorist: takes $3.85 US$ to a bank and exchanges for $\frac{3.85}{0.70} = 5.50 CDN$

Immediate profit: $0.50 CDN$

This is an investment that gives immediate positive reward, with no future obligations: Type A arbitrage
11.6 "BOGO" arbitrage example:

Shoe Store has a "Buy one, get one for 50% off" sale.

Investor finds two people who each want 1 pair of shoes.

Investor borrows \((P + 0.5P)\) from bank; \(P = \text{price of a pair of shoes}\)

- pays \(1.5P\) for two pairs.

- sells each pair, for \(P\);
  
  now has \(2P\) in hand.

- repays bank \(1.5P\)

Immediate profit: \(2P - 1.5P = 0.5P\).

Also: type A arbitrage,

an investment that has no (or negative) initial cost,

but that has positive probability of positive payoff

in future, is called type B arbitrage.
Linear pricing

Let \( d \) be a security with price \( P \).

Let \( 2d \) be the security that always pays twice what \( d \) pays.

Let \( Q = \text{price of } 2d \).

If \( Q > 2P \):
- borrow \( 2P \) from bank
- use \( 2P \) to buy two copies of \( d \)
- bundle two copies into \( 2d \)
- sell \( 2d \) for \( Q \):
  - now have \( Q \) in hand
- pay back bank with \( 2P \)
- immediate profit \( (Q-2P) > 0 \)

If \( Q < 2P \):
- borrow \( Q \)
- purchase \( 2d \) with \( Q \)
- break up (unbundle) \( 2d \) into two copies of \( d \)
- sell each copy for \( P \):
  - now have \( 2P \) in hand
- pay back bank with \( Q \)
- immediate profit \( (2P-Q) > 0 \).

If assume no arbitrage is possible,

must be: both \( Q > 2P \) false, and \( Q < 2P \) false.

\[ Q \leq 2P \quad \text{and} \quad Q \geq 2P \implies Q = 2P. \]
For securities that can be divided into fractions, similar argument:

If \( d_1 \) = security with price \( P_1 \),
\[ d_2 = \text{security with price } P_2, \]
then: price of \( \alpha d_1 + \beta d_2 \) must be \( \alpha P_1 + \beta P_2 . \)

(Linear Pricing)