20.0 Historical volatility
Ch. 20: Historical Volatility.

Assume data: asset $S_i$ at $t_i$, $t_i = i \Delta t$

(large # points).

form $U_i = \ln \left( \frac{S_i}{S_{i-1}} \right)$, $i = 1, 2, 3, \ldots$

compute: $U_1 = \ln \left( \frac{S_1}{S_0} \right)$, $U_2 = \ln \left( \frac{S_2}{S_1} \right)$ etc. $\varepsilon_i$ is std. normal r.v.

Expect from model $S_i = S_{i-1} e^{(\mu - \frac{\sigma^2}{2}) \Delta t + \sigma \varepsilon_i \sqrt{\Delta t}}$

expect $U_i$: normal r.v., mean $(\mu - \frac{\sigma^2}{2}) \Delta t$, std. dev. $\sigma \sqrt{\Delta t}$.

From data, compute

$$q_M = \frac{1}{M} \sum_{i=1}^{M} U_i, \quad \varepsilon_M^2 = \frac{1}{M - 1} \sum_{i=1}^{M} (U_i - q_M)^2$$
Can use \( \text{ln} \) to provide c.i. for approx. of \( (n - \frac{6}{2}) \)

Can also approx. \( 6 \sqrt{\Delta t} \approx \text{ln} \)

\[ 6 \approx \text{lnm} / \sqrt{\Delta t} \]

For large \( m \), and approx. 95\% c.i.

for approx. of \( 6^2 \) by \( \frac{\text{lnm}^2}{\Delta t} \)

\[ \Rightarrow 6^2 \approx \frac{1.96 \nu}{\text{lnm}} \]

where \( \nu^2 = \text{var of} \ (\hat{\nu} - E(\hat{\nu}))^2 = 26^+ \)

\[ \hat{\nu} = \frac{\nu}{\sqrt{\Delta t}} \]

exer. bt 20.3