2.5 Put-call parity
Switch to Higham:

Recall def'ns of puts, calls from 458.

Relationship between (Euro) calls & puts values.

Put-Call Parity.

Let $\Pi_A$: one call option + P.V. of strike $K$ invested in bank.

$D_{0,T} = \text{discount factor for } (0, T)$:

Switch to continuous compounding: $D_{0,T} = e^{-rT}$, $r =$ rate.

Let $\Pi_B$: one put option + one unit of asset $S$. 
At $t = T$: $\Pi_A$ is worth

$$\max (S(T) - K, 0) + (K \Delta S_T) / \Delta t, T = \begin{cases} S(T), & S(T) \geq K \\ K, & S(T) < K \end{cases}$$

And at $t = T$, $\Pi_B$ is worth

$$\max (K - S(T), 0) + S(T) = \begin{cases} S(T), & S(T) \geq K \\ K, & S(T) < K \end{cases}.$$

Since portfolios produce same outcome at expiration, expect values at $t = 0$ of portfolios should be equal.

If $\Pi_A$ worth more than $\Pi_B$ at $t = 0$:

Then: sell $\Pi_A$ & buy $\Pi_B$ at $t = 0$:

(sell call option, and (buy put option
borrow cash) and (sale stock)

Instantaneous profit: no arbitrage $\Rightarrow \Pi_A(0) \leq \Pi_B(0)$.

(At $t = T$, $\Pi_A = \Pi_B$)
If: $\Pi_B$ worth more than $\Pi_A$ at $t = 0$, then: buy $\Pi_A$ & sell $\Pi_B$ at $t = 0$

(buy call option, and (sell one put option
lead cash)) sell one share).

Instantaneous profit, and at $t = T$, $\Pi_A - \Pi_B$:

no arbitrage $\Rightarrow \Pi_B(0) \leq \Pi_A(0)$.

- $\Pi_A(0) = \Pi_B(0)$.

$C + K \Delta_{0,T} = P + S$, at $t = 0$.

$C = C(t; K)$, $P = P(t; K)$; $S = S(t)$.

$C(0; K) + K e^{-rT} = P(0; K) + S(0)$.

$t = 0$ is not special:

$C(t; K) + K e^{-r(T-t)} = P(t; K) + S(t)$, any $0 \leq t < T$. 