19.3 Lookback options
Path dependent options natural with a call
Barrier options saw down & out last day.
also up and in, up and out, down and in natural with put.

Parisian barrier options:
to be knocked out, asset must cross a barrier in
and stay across for a period of time.

Lookback options:
Value of option depends on previous values of asset:

\[
\text{Set } S_{\text{max}} = \max_{0 \leq t \leq T} S(t), \quad S_{\text{min}} = \min_{0 \leq t \leq T} S(t)
\]
Asian Options:

Use the average of the asset in place of \( S_{\text{max}}, S_{\text{min}} \) in floating strike option

Continuous average: \[ \frac{1}{T} \int_{0}^{T} S(t) \, dt \]

Or: \[ \frac{1}{N} \sum_{j=1}^{N} S(t_j) = S_{\text{avg}}. \]

Average price call:
\[
\text{Payoff} = \max \left( S_{\text{avg}} - K, 0 \right)
\]

Average price put
\[
\text{Payoff} = \max \left( K - S_{\text{avg}}, 0 \right).
\]

Or: make \( S_{\text{avg}} \) into the strike price.
\[
\Rightarrow \text{Call: payoff} = \max \left( S(T) - S_{\text{avg}}, 0 \right)
\]
\[
\text{Put payoff} = \max \left( S_{\text{avg}} - S(T), 0 \right).
\]
Fixed strike lookback call:

Fix $K$. Payoff = max ($S_{\text{max}} - K$, 0).

Fixed strike lookback put:

Payoff = max ($K - S_{\text{min}}$, 0).

Floating strike lookback call:

Payoff = max ($S(T) - S_{\text{min}}$, 0).

Floating strike lookback put:

Payoff = max ($S_{\text{max}} - S(T)$, 0).

\[ S_{\text{max}} \geq S(T) \quad \text{so} \quad \text{payoff} \geq \text{payoff for } C_{\text{euro}} \]
\[ S_{\text{min}} \leq S(T) \quad \text{so} \quad \text{payoff} \geq \text{payoff for } P_{\text{euro}} \]

use $S_{\text{min}}$ as strike.

use $S_{\text{max}}$ as strike.
Can modify B-S with continuous averages

Let \( I(t) = \int_0^t S(\tau) \, d\tau \).

Then \( S_{avg} = \frac{1}{T} \int_0^T S(\tau) \, d\tau \), \( \frac{1}{T} \int_0^T S(\tau) \, d\tau = \frac{I(T)}{T} \).

The option price becomes a function of 3 vars: \( f(S, I, T) \).

Modified B-S

\[
\frac{\partial f}{\partial t} + S \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = r f.
\]

New

Now: B.C.'s in terms of \( I, S, t \): see above

Call: \( f(S, I, T) = \max \{ S - \frac{1}{T} I, 0 \} \)

Put: \( f(S, I, T) = \max \{ \frac{1}{T} I - S, 0 \} \).

Need to solve numerically (PDE methods),
or by Monte-Carlo simulation.
Bermuda is between the U.S. & Europe Amer.

Bermuda option:
  may be exercised at certain discrete times
  (exp. once each month)
  up to expiration.

To value - use binomial lattice; like Amer. style option,
  only apply possibility of exercising,
  at discrete times.