1a) \( f(x) = e^x + e^{-x} \)  \( f'(x) = e^x - e^{-x} \)
Set \( f'(x) = 0 \Rightarrow e^x - e^{-x} = 0 \Rightarrow e^x = e^{-x} \Rightarrow x = -x \Rightarrow x = 0 \)
For \( x < 0 \), \( e^x < 1 \) and \( e^{-x} > 1 \), therefore \( f'(x) = e^x - e^{-x} < 0 \)
So, \( f(x) \) is decreasing for all \( x \) less than the global min, 0.
For \( x > 0 \), \( e^x > 1 \) and \( e^{-x} < 1 \), therefore \( f'(x) = e^x - e^{-x} > 0 \)
So, \( f(x) \) is increasing for all \( x \) greater than the global min.
Therefore, \( f(x) \) is unimodal. \( f(0) = e^0 + e^{-0} = 1 + 1 = 2 \)

1c) \( f(x) = 2x^4 + x \)  \( f'(x) = 8x^3 + 1 \)
Set \( f'(x) = 0 \Rightarrow 8x^3 + 1 = 0 \Rightarrow x^3 = -\frac{1}{8} \Rightarrow x = -\frac{1}{2} \)
For \( x < -\frac{1}{2} \), \( 8x^3 < -1 \), therefore \( f'(x) = 8x^3 + 1 < 0 \)
So, \( f(x) \) is decreasing for all \( x \) less than the global min, \(-\frac{1}{2}\).
For \( x > -\frac{1}{2} \), \( 8x^3 > -1 \), therefore \( f'(x) = 8x^3 + 1 > 0 \)
So, \( f(x) \) is increasing for all \( x \) greater than the global min, \(-\frac{1}{2}\).
Therefore, \( f(x) \) is unimodal. \( f(-\frac{1}{2}) = 2(-\frac{1}{2})^4 - \frac{1}{2} = -0.375 \)

2a) \( f(x) = \cos x \)  \( f'(x) = -\sin x \)
Set \( f'(x) = 0 \Rightarrow -\sin x = 0 \Rightarrow \sin x = 0 \Rightarrow x = n\pi \), \( n \) = integer
On the interval \([3, 4]\), \( n = 1 \) gives \( x = \pi \), so \( f_{\text{min}} = f(\pi) = \cos \pi = -1 \)
For \( 3 < x < \pi \), \( \sin(x) > 0 \), so \( f'(x) < 0 \) and \( f(x) \) is decreasing.
For \( \pi < x \leq 4 \), \( \sin(x) < 0 \), so \( f'(x) > 0 \) and \( f(x) \) is increasing.

2c) \( f(x) = x^3 + 6x^2 + 5 \)  \( f'(x) = 3x^2 + 12x \)
Set \( f'(x) = 0 \Rightarrow 3x^2 + 12x = 0 \Rightarrow x^2 + 4x = 0 \)
\( x = \frac{-4 \pm \sqrt{16 - 0}}{2} = \frac{-4 \pm 4}{2} = 0, -4 \)
For \(-5 < x < -4\), \( f'(x) > 0 \), i.e., \( f(x) \) is increasing.
For \(-4 < x < 0\), \( f'(x) < 0 \), i.e., \( f(x) \) is decreasing.
For \(0 < x \leq 5\), \( f'(x) > 0 \), i.e., \( f(x) \) is increasing.
Therefore, \( x = -4 \) is a maximum and \( x = 0 \) is a minimum.
The minimum could be at the beginning of the interval, but \( f(-5) = 30 > f(0) = 5 \), so the minimum of \( f \) occurs at \( x = 0 \).
Problem 13.1 CP 1(a)

I solved this problem by editing gss_plot.py. It is clear from the plot that the function is unimodal on the length-one interval [0, 1] since the function is first decreasing and then increasing.

I edited the function as follows:

```python
return 2.*x**4 + 3.*x**2 - 4.*x + 5.
```

I also added a stopping criteria by augmenting the while condition:

```python
newguess = b
oldguess = a
while abs(newguess-oldguess)>1e-6:
    oldguess = newguess
    newguess = (a+b)/2
```

Here is the result of running the code:

![Graph](image)

**Golden section search for function from 13.1 CP 1(a)**

<table>
<thead>
<tr>
<th>step</th>
<th>a</th>
<th>x1</th>
<th>x2</th>
<th>b</th>
<th>xguess</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>0.38196601125</td>
<td>0.61803398875</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>21</td>
<td>0.499954321213</td>
<td>0.499979571825</td>
<td>0.499995177562</td>
<td>0.4999954321213</td>
<td>0.499979571825</td>
</tr>
<tr>
<td>22</td>
<td>0.499979571825</td>
<td>0.499995177562</td>
<td>0.500004822438</td>
<td>0.500004822438</td>
<td>0.500004822438</td>
</tr>
</tbody>
</table>

(xmin, ymin) is approximately (0.499992197132, 3.875000000037)

Since I know the exact answer is ½, it is clear that the approximate xmin has five correct digits.
Problem 13.1 CP 5

I solved this problem by editing neldermead.py.
I edited the numpy array print options in order to see 16 digits of each step:

```python
set_printoptions(precision=16)
```

Here is the relevant result of running the code:

```python
iter = 61
xbar = [1.2088175899558531 1.2088175929781708]
```

During this step: yr<yn? No; yr<yn? No; yic<yn? Yes; accept inside contract

Unsorted data at end of step

<table>
<thead>
<tr>
<th>function values y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.205427886650958</td>
</tr>
</tbody>
</table>

vertices x

| [1.2088175915084491 1.208817588403257 1.208817586653932] |
| [1.2088175914157684 1.2088175945405735 1.2088175919470627] |

Sorted data at end of step

<table>
<thead>
<tr>
<th>function values y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.205427886650958</td>
</tr>
</tbody>
</table>

vertices x

| [1.2088175915084491 1.208817588403257 1.208817586653932] |
| [1.2088175914157684 1.2088175945405735 1.2088175919470627] |

... After 69 steps, neldermead gives vertices xx=

| [1.2088175899689666 1.208817586257596 1.20881758980034194] |
| [1.2088175925297932 1.20881759249025391 1.208817592365871] |

and function values yy=[0.205427886650958 0.205427886650958 0.205427886650958]

After 61 iterations, with initial guess (1,1) and search radius 1, the solution for (x,y) has 8 correct digits (1.208817589, 1.208817589). After 69 iterations, the solution has no more consistent digits, implying that this is the maximum accuracy obtainable. Using initial guess (1.2088, 1.2088) without changing the radius does not reduce the number of iterations necessary to achieve this accuracy. Using either initial guess (1,1) or initial guess (1.2088, 1.2088) with radius 0.1, the solution is found after 56 iterations. Using initial guess (1.2088, 1.2088) with radius 0.0001, the solution is found after only 30 iterations.